CURRENT ACCOUNT IMBALANCES, THRIFTINESS AND THE REAL EXCHANGE RATE IN A GROWING ECONOMY

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Abstract

This paper analyses the theoretical relevance of the dynamical aspects of growth on the discussion about the observed positive correlation between per capita real income and real exchange rates. With this purpose, we develop a simple exogenous growth model where the internal, external and intertemporal equilibrium conditions of a typical macroeconomic model are imposed; this last one through the inclusion of a balanced growth path for the foreign assets accumulation. The main result under this consideration is that the relationship defended by the Balassa-Samuelson hypothesis is no more so straightforward. In our particular approach, the mentioned bilateral relationship depends on a parameter measuring thriftiness in the economy. Therefore, the probability of ending up with a positive relationship between growth and real exchange rates –as the classical economic theory predicts– will be higher when the economy is able to maintain a minimum saving ratio. Moreover, given that our model considers a simple Keynesian consumption function, some explosive paths can also be possible.

Keywords: Balassa-Samuelson hypothesis, foreign debt, growth, macroeconomic approach, per capita income, real exchange rate, thriftiness

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1. Introduction

The purchasing power parity (PPP hereafter) assumes a constant level to which the real exchange rate would eventually converge in the long run. This is a classical reference in the literature on real exchange rates that however, and despite the attractiveness of its simplicity, has presented many problems in order to be empirically corroborated. In fact, the generalised result of a great deal of available research on the topic is precisely the non-stationarity of real exchange rates. This way, and against the PPP prevision, data seem to predict a systematic price level deviation among countries, which is in some degree independent of the economic integration between them.

An additional drawback regarding the PPP assumption is the simplification that it introduces on the price level behaviour of tradable and nontradable goods. This inconvenience is in fact related to the consideration of a long run stationary real exchange rate, since it necessarily involves that the tradable and nontradable sectorial price levels evolve similarly over time. However, given that nontradables are not subject in reality to the international competition, there is no reasonable argument that can justify this assumption of a similar sectorial evolution.

This kind of assumptions, extremely simplistic in many occasions, is what has led many authors in general and us in particular to the conviction that the real exchange rates do not have to be necessarily constant in the long run but, on the contrary, that there are real factors which affect and determine the real exchange rate behaviour[1].

There is also an interesting empirical regularity very frequently underlined by the international economic research and which highlights not only that the real exchange rate is a non-stationary variable, but that there is also a strong positive relationship between the real exchange rate and the per capita income level[2]. In this approach the two predominant competing theories underlying the role of the per capita income level are a supply-oriented approach, known as the Balassa-Samuelson hypothesis, and a demand-oriented one that, alternatively to the previous proposal, avoids the assumption of a perfectly elastic relative demand function. Empirical evidence on the Balassa-Samuelson model can be found, for instance, in Asea and Mendoza (1994) for a group of fourteen OECD countries and in Micossi and Milesi-Ferretti (1994) for the context of the EMS. Regarding the demand side approach the reader can refer to Neary (1988), Bergstrand (1991) or De Gregorio et al (1994).

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[1] In a small economy the terms of trade are commonly assumed an exogenous variable, so the real exchange rate results directly proportional to the ratio of relative prices between tradable and nontradable goods. Given that this will be a crucial assumption in the whole paper, we will refer indifferently to the real exchange rate or to the ratio of relative prices.

[2] Evidence on this assertion can be found for instance in Kravis and Lipsey (1983, 1987, 1988).

In this paper we go one step further and work on a third hypothesis, complementary in fact to the previous supply and demand-oriented ones, but that underlines instead the intertemporal restrictions that a process of economic growth necessarily imposes on the economy. This research is indeed related to the well-known transfer problem in international economics that, very recently, has been taken up again in order to highlight the influence that the net foreign asset position has on the real exchange rate determination.

As Obstfeld and Rogoff (1995) or Lane and Milesi-Ferretti (2000) underline, there is a persistence effect derived from the financing of the growth process that cannot be forgotten when analysing the foundations of the real exchange rate behaviour. It is well known that a period of high factor productivity is going to be characterised by capital inflows that, although being able to finance the new investment requirements, would also end up generating a stock of external liabilities that will need to be serviced in the near future. Countries in that case will have to run surpluses large enough so as to serve the net external payments, being a more depreciated national currency in this context finally required. Only the capital accumulation associated to the growth period could eventually reverse the process, giving the country the opportunity to end up as an asset holder.

Formally, the specific literature concerned with the intertemporal aspects of the exchange rate equilibrium has been mainly associated to the macroeconomic balance approach, which has insistently underlined the role of the intertemporal balance condition in the context of a growing economy. Within the current research in this area, and particularly in the context of developed economies, it has stood out the FEER approach of Williamson (1994), the DEER approach of the group of economists of the IMF[3] and the NATREX approach of Stein (1990, 1994) and Stein, Allen and associates (1995). For the case of developing economies, the works of Edwards (1988) and Eldabawi (1994) can be alternatively underlined. However, from the point of view of our research, the main problem of all this literature is that it has been designed following a pure empirical approach. None of them have devoted any special effort to formally analyse the determinants of the relationship between growth and real exchange rates. The theoretical analysis of this economic phenomenon, including the intertemporal aspects of equilibrium, is therefore the main contribution of this paper to the existent literature on the topic.

Following this reasoning, the current paper investigates how the classical theoretical approaches analysing the positive relationship between the income level and the real exchange rate can be enriched with the assumption that, in the long run, a balanced growth condition for the foreign asset accumulation has to be necessarily considered. With this objective, we develop a simple exogenous growth model where it is imposed the internal and external balance condition, plus the appropriate intertemporal asset restriction. The main conclusion of our particular small-

[3] For this approach see for instance the works of Artis and Taylor (1993) or Bayoumi, Clark, Symansky and Taylor (1994).

scale model is that the result defended by the Balassa-Samuelson hypothesis is no more so straightforward, due to the existence of a thriftiness parameter that influences the bilateral relationship of real exchange rates and growth. In particular, our main result is that the probability of having a positive relationship between the two variables –as the classical economic theory predicts– will be higher when the economy is able to maintain a minimum saving ratio. In addition, as our model considers a simple Keynesian consumption function, some explosive paths can be possible.

The paper is organised as follows. Next section goes briefly through the traditional demand and supply hypothesis to explain the classical relationship between the relative price index and income. Section three presents the main body of the work, being here where the intertemporal equilibrium model is fully developed. Formally, this equilibrium model stands out for the precise characterisation of the economic growth process, since only in this way the long run effects of the accumulation in the stock variables can be taken into account. Given that investment requirements are usually different from the volume of internal savings, the existence of transference of resources between complementary economies will be the most common result in an international context of openness. In particular, there are the conditions associated to this international loan that will determine the long run evolution of the real exchange rate. This is the reason why in this model the convergence process, as well as its stability, is considered a main element in the real exchange rate determination. Finally, section four summarises the main conclusions of the paper.

2. Fundamental Determinants of the Relative Price Level

2.1 The Supply Approach

Under this supply-oriented approach we will simply underline the implications of the Balassa-Samuelson (BS) hypothesis[4]. Formally, the BS model[5] concludes that the greater the productivity of tradables in relation to nontradables, the more appreciated the real exchange rate will be in a particular economy. The importance of this hypothesis is that it can easily explain the stronger currencies observed in economies with higher income levels, given the consideration that these more developed countries present higher differentials in the level of relative productivity. Let us briefly revise the results of this proposition in analytical terms.

We consider the case of a small economy with two sectors, tradables and nontradables, each one with a Cobb-Douglas production function like the following,

[4] The Heckscker-Ohlin model, although less known in the literature, is also a supply-sided approach that arises as an alternative to the Balassa-Samuelson hypothesis. This model bases its reasoning not in the role of the relative productivity of factors, but on the different factor intensity according to the degree of economic development. Bhagwati (1984) is a good reference to see an application of it.

[5] Developed from separated contributions of Balassa (1964) and Samuelson (1964).

$$Y_i = A_i K_i^{1-\alpha_i} L_i^{\alpha_i} \quad i = T, N \tag{1}$$

where T and N refer to the sector of tradables and nontradables respectively, Y is the level of production, K is the capital stock, L is the labour force and A_i is the global productivity factor in sector i.

In the case that the production factors are homogeneous, in equilibrium their costs will equalise across sectors. Therefore, assuming a competitive context and profit maximising firms, we can optimise with respect to K and L in order to obtain the equilibrium price levels of this economy. These appear in expressions (2) and (3) respectively,

$$P_T = \frac{1}{A_T} \frac{\omega^{\alpha_T} r^{1-\alpha_T}}{\alpha_T^{\alpha_T} (1-\alpha_T)^{1-\alpha_T}} \tag{2}$$

$$P_N = \frac{1}{A_N} \frac{\omega^{\alpha_N} r^{1-\alpha_N}}{\alpha_N^{\alpha_N} (1-\alpha_N)^{1-\alpha_N}} \tag{3}$$

being P_T the price level of tradables, P_N the price level of nontradables and ω and r the cost of labour and capital factors respectively.

Assuming that the tradable sector is subject to international competence, it will occur that $P_T = E \times P_T^*$ (being E the nominal exchange rate) so that P_T can be considered as given. Hence, any relative improvement in the productivity of tradables will go to wages and, eventually, to the price of nontradables.

Specifically, if we consider that $P = P_N / P_T$, then P is defined as follows,

$$P = \frac{\omega^{\alpha_N} r^{1-\alpha_N}}{\alpha_N^{\alpha_N} (1-\alpha_N)^{1-\alpha_N}} \frac{1}{A_N P_T} \tag{4}$$

With P_T being an exogenous variable, changes in A_T will translate completely into wage changes. In that case we can find ω in (2) and substitute it in (4) to obtain P as a function of A_N, A_T and the corresponding exogenous variables,

$$P = \frac{A_T^{(\alpha_N/\alpha_T)}}{A_N} \left[\left(\frac{\alpha_T}{\alpha_N} \right)^{\alpha_N} \frac{(1-\alpha_T)^{(1-\alpha_T)(\alpha_N/\alpha_T)}}{(1-\alpha_N)^{1-\alpha_N}} \left(\frac{P_T}{r} \right)^{(\alpha_N-\alpha_T)/\alpha_T} \right] \tag{5}$$

Finally, working with (5), we can easily derive the growth rate of the relative price index as a function of the respective productivity factor rates in both sectors, as it appears in (6),

$$\hat{P} = \frac{\alpha_N}{\alpha_T}\hat{A}_T - \hat{A}_N \qquad (6)$$

The conclusion under this approach is therefore that, in the context of small economy with given international price indexes and perfect factor mobility, a rise in the relative price ratio follows an improvement in the tradable good sector productivity.

2.2 The Demand Approach

The demand-oriented approach introduces the role of income in the analysis of real exchange rates through the different effect that it has on the income elasticity of the relative demand function. Under this approach the national currency will tend to appreciate whenever the income elasticity of nontradables is greater than the corresponding to tradables. Formally, this relationship between elasticities and the real exchange rate is carefully explained below.

In this approach it is assumed that agents incorporate a minimum subsistence level in their consumption of tradables, so we can get demand functions with income elasticities less than one for tradables and greater than one for nontradables respectively. In this case, a rise in income will lead to a greater increase in the wish to consume nontradables than tradables, what will cause a shift in the relative demand function to the right. However, the effect of this shift on demand will still depend on the supply side behaviour. Under the triple assumption of perfect competence in the good and factor markets, the fulfilment of the law of the single price and the existence of perfect capital mobility, a relative supply function put in the same terms that the demand function will be completely horizontal at the level determined in (5). In that case, the effect that the rise in income has on the relative demand function will simply lead to changes in the composition of production, but not in the relative price level. However, if we lose any of the previous assumptions, what is in fact the most common situation, the supply function will end up with a positive slope and a shift in relative demand will necessarily provoke a higher relative price index in a near future.

Note that this demand approach is in reality a complement of the previous supply-oriented one. Notwithstanding, for the case of an open economy, the external equilibrium condition should be also taken into account. The inflow and outflow of financial resources influences to a great extent the nominal exchange rate, so that the price level of tradables expressed in national currency is consequently affected and so is the supply function. This role of the external sector, constituting in fact our main contribution to the analysis of equilibrium, is thoroughly developed in the next section.

3. The Real Exchange Rate and the Income Growth Process: A Macroeconomic Equilibrium Approach

In this section, we propose a dynamic model where both the internal and external equilibrium

conditions are going to characterise the long run of the national economy. The approach will particularly underline the effect that over the long run real exchange rate would have the intertemporal restriction derived from the external balance equilibrium. Formally, we will start identifying the peculiarities of this balance of payments equilibrium, as well as that of the national demand and supply conditions. Subsequently, the properties of the steady state will be carefully described.

3.1 The Balance of Payments Equilibrium

We know that the disposable income, determined by the national income plus the net transferences of the public and foreign sectors, is distributed between consumption and savings. However, in order to simplify we are going to consider both the absence of a public sector and that the net payments coming from the foreign sector will be merely composed of the net payment of interests from the rest of the world. This way, it is easy to obtain that the total production (Y) distributes between private consumption (C), private savings (S) and the service of the net foreign debt[6] (DS), such as it appears in expression (7),

$$Y = C + S + DS \tag{7}$$

Since the components of production are consumption (C), investment (I) and trade balance[7] (TB), making use of (7) the external equilibrium condition is determined as follows,

$$TB - DS + I - S = 0 \tag{8}$$

Alternatively, given the equality in (9),

$$TB - DS = CA = -KB = -\Delta D = S - I \tag{9}$$

where TB and KB are the trade and capital balances respectively, CA is the current account and D represents the stock of foreign debt, (8) can be in addition represented by the more informative expression (10),

$$TB + \underbrace{\left[\underbrace{I - S}_{\Delta D} - DS\right]}_{\text{Foreign Net Resources}} = 0 \tag{10}$$

Equation (10) says that, adopting the standard assumption of a negative relationship between

[6] This amount must have the opposite sign in the case that the national economy would posses foreign assets instead of foreign debt.

[7] Remember that we have let aside the public sector.

the trade balance and the real exchange rate[8], the greater the inflow of foreign resources the stronger the currency of a country will be. Our aim is therefore to determine the conditions that in the steady state would characterise a net inflow of resources.

3.2 The Demand Equilibrium Condition

In the case of an open economy with absence of public sector, the aggregate demand function is simply determined by the addition of the consumption, investment and trade balance components, as expression (11) shows,

$$Y = C + I + TB \tag{11}$$

Regarding the trade balance, from (10) it can be specifically defined as in equation (12), where it appears as a function of the level of net foreign debt,

$$TB = (r - \hat{D})\ D \tag{12}$$

Including the result of (12) into equation (11), it is then obtained the expression (13) that represents an alternative to (11) in the definition of the aggregate demand function,

$$Y = C + I + (r - \hat{D})\ D \tag{13}$$

In per capita terms, equation (13) can be additionally simplified to expressions (14) or (15), depending on whether the relationship given by $\hat{D} = \hat{d} + n$ is finally taken into account,

$$y = c + i + (r - \hat{D})\ d \tag{14}$$

$$y = c + i + \left[(r - n) - \hat{d}\right] d \tag{15}$$

Lower case letters indicate per capita terms and *n* is the growth rate of the working population force. This one, without loss of generality, can be approached by the whole population.

Reformulating now expression (15) in growth rates and grouping conveniently, it is finally obtained the aggregate demand equilibrium condition given by (16),

$$0 = (\hat{c} - \hat{y})\ \frac{c}{y} + (\hat{i} - \hat{y})\ \frac{i}{y} + \left[(r - n) - \hat{d}\right](\hat{d} - \hat{y})\ \frac{d}{y} \tag{16}$$

[8] Defined such that an increment means a real appreciation of the national currency.

Note that if the economy had foreign assets (named F in absolute terms and f in per capita terms) instead of foreign debt, it would follow that $f = -d$ and $\hat{f} = \hat{d}$. In that case, expression (16) would become the following,

$$0 = (\hat{c} - \hat{y})\frac{c}{y} + (\hat{i} - \hat{y})\frac{i}{y} + \left[\hat{f} - (r - n)\right](\hat{f} - \hat{y})\frac{f}{y} \tag{17}$$

Equations (16) and (17) will alternatively represent the demand side behaviour of the economy under analysis.

3.3 The Supply Equilibrium Condition

Regarding the supply equilibrium condition, we will start with the characterisation of the national production function that, in particular, is assumed as follows,

$$Y = A\,F(K,L) \quad F'_K, F'_L > 0 \;;\; F''_{KK},\ F''_{LL} \leq 0 \tag{18}$$

where K is the capital stock, L is the labour force and A a global productivity factor.

Assuming constant returns to scale[9], the production function is easily redefined as in (19) where variables are expressed in per capita terms,

$$y = A\,f(k) \tag{19}$$

Working in growth rates from (19), the equation (20) can be easily derived. This one relates the income growth rate with the rate of technological progress and the growth rate of the per capita stock of capital,

$$\hat{y} = \hat{A} + \gamma\hat{k} \;;\; \gamma = \frac{f'_k}{f(k)}k \tag{20}$$

This equation (20) is then used to characterise the supply side behaviour of our growing economy.

3.4 The Steady State

The steady state condition will satisfy the equations (10), (16) or (17), and (20). Next, we carefully analyse what these equilibrium conditions would imply over the long run levels of

[9] It is standard practice to assume constant returns to scale as this is commonly corroborated by the empirical analysis.

consumption, investment, trade balance, income and net foreign debt.

With regard to investment, we know that in economic terms the whole quantity of gross investment is devoted both to raise the capital stock and to pay its total amount of depreciation, that is to say,

$$I = \Delta K + \theta K \tag{21}$$

where θ is the depreciation rate.

Alternatively, in per capita terms equation (21) can be reformulated as follows,

$$i = \left(\hat{K} + \theta\right) k \tag{22}$$

From (22), and given that $\hat{K}$ and θ are stable in the steady state, the growth rates of both gross investment and capital stock in per capita terms should converge in equilibrium, i.e. $\hat{i} = \hat{k}$.

Regarding the demand side behaviour, we see that for the equation (16) to be fulfilled, it is a necessary equilibrium condition that $\hat{c} = \hat{i} = \hat{y} = \hat{d}$ or, alternatively, that $\hat{c} = \hat{i} = \hat{y} = \hat{f}$, so the condition $\hat{i} = \hat{k} = \hat{y}$ will be also satisfied. Once these relationships are included into (20), we can get the final expression (23) for the per capita income growth rate,

$$\hat{y} = \frac{1}{1-\gamma}\hat{A} \tag{23}$$

From (23) it is easily deduced that in equilibrium the per capita income will grow at a greater rate than the technological progress. The reason is that the technological factor is influencing income both directly and indirectly, this last one through its effect on the capital stock.

Also regarding equation (23), it is important to notice that for the result to be coherent, the parameter γ must be less than one, i.e. $f'_k < f(k)/k$. This would imply that in the steady state the marginal productivity of capital has to be lower than its mean productivity, which is always satisfied if the production function is a concave one.

Finally, we need still to conclude about the trade balance behaviour. In particular, from (12) it is easily deduced that, in equilibrium, the ratio TB/D is going to be stable over time, so in a steady state situation the trade balance growth rate should also equalise the growth rate of the stock of net foreign debt.

The next step should be then to characterise the convergence towards equilibrium. Our aim in

this respect is to find out which are the circumstances that, in a steady state position, are going to determine the possible net inflow or outflow of foreign financial resources.

3.5 The Convergence to Equilibrium and its Fundamental Determinants

We know from the previous section that in equilibrium $\hat{c} = \hat{i} = \hat{k} = \hat{d}\,(\text{or } \hat{f}) = \hat{tb} = \hat{y} = \eta\,\hat{A}$. However, without loss of generality we can assume that $\hat{A} = 0$. The steady state in that case would simplify to one where $\hat{C} = \hat{I} = \hat{K} = \hat{D}\,(\text{or } \hat{F}) = \hat{TB} = \hat{Y} = n$, so in per capita terms these steady state growth rates will clearly approach to zero.

Let us start this analysis through the study of the investment and capital stock accumulation to continue next with the convergence and steady state properties of the rest of important variables in this model, as is the case of income, savings, foreign debt and real exchange rates.

3.5.1 Investment Decisions and the Capital Stock Dynamics

In a standard economy, the first stages of economic growth are usually characterised by a small accumulation of capital stock, so its marginal return becomes high enough as to promote a large investment rate. However, in the long run the arrival of diminishing returns in the production factor would cause that the capital stock growth rate decreases over time, until the steady state is finally reached and $F'_K(K,L) = r$. Let us analyse next the analytical implications of the process.

Expression (24) shows the particular determinant factors in the evolution of F'_K over time,

$$\Delta[F'_K(K,L)] = F''_{KK}\Delta K + F''_{KL}\Delta L = F''_{KK}K\left(\hat{K} + \frac{F''_{KL}}{F''_{KK}}\frac{L}{K}n\right) \tag{24}$$

However, using the assumption of constant returns to scale we can still simplify (24) into a more intuitive expression. Specifically, with constant returns to scale the following condition is necessarily satisfied,

$$F(K,L) = (F'_K)K + (F'_L)L \tag{25}$$

If we derive (25) with respect to K and reorganise[10], the following relationship can be easily obtained,

$$\frac{L}{K} = -\left(\frac{F''_{KK}}{F''_{KL}}\right) \tag{26}$$

[10] Note that the labour force is by definition independent of the capital stock, so dL/dK=0.

Substituting next (26) into (25), the equation (27) is finally determined,

$$\Delta[F'_K(K,L)] = (\hat{K} - n)\, F''_{KK} K \tag{27}$$

This expression can be used to analyse the dynamic behaviour of the investment and capital stock variables.

With respect to the long run equilibrium, it is already known that an optimising agent is encouraged to invest whenever $F'_K > r$, so it should occur that $\Delta(F'_K) < 0$ over time. Following equation (27), we can see that the fact that F''_{KK} is negative is what guarantees this dynamic convergence to equilibrium. Additionally, since the steady state is reached when $\Delta(F'_K) = 0$, and given the condition $\hat{K} = (I/K) + \theta$, it should also occur in equilibrium that $\hat{I} = \hat{K} = n$; a result that is coherent with the previous conclusions about the steady state characterisation.

Coming back to expression (27), we can see that the circumstance of a decreasing marginal productivity of capital is obtained[11] whenever $\hat{K} > n$. The conclusion is therefore that in the case of a growing economy, the first stages of growth are usually characterised by a high capital stock growth rate along with a decreasing trend in the long run. Formally, this would imply that $\Delta(\hat{K})$ has to be negative for convergence to take place. In particular, we can use expression (28) to analyse what the previous requirement would imply on investment,

$$\Delta\hat{K} = \frac{\Delta^2 K}{K} - \hat{K}^2 = (\hat{I} - \hat{K})\,\hat{K} \tag{28}$$

According to (28), for $\hat{K}$ to diminish in the process of convergence, investment must be such that $\hat{I} < \hat{K}$. Unfortunately, the knowledge of this condition does not allow us to specify if the per capita level of investment increases or decreases over time in its trajectory to the steady state position; both of these circumstances might happen in fact. We cannot forget at this point that this work does not enter in detail into the concrete dynamics of the investors' decisions[12].

In the same way, the stability of the process can be formally proved through the analysis of the capital stock dynamic equation. To start with, we can reformulate identity (21) in per capita terms as follows,

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[11] Remember that $F''_{KK} < 0$.

[12] It could be possible, for instance, that in the first stages of growth the differential between F'_K and r provoked a rapid increase in investment, so that the latter increased in per capita terms. But, as the marginal productivity of capital and the interest rate got together, investment could slow down and even fall down in per capita terms. However, this is not the only way in which this dynamics can be characterised.

$$dk = i - (\theta + n)\,k \tag{29}$$

Or alternatively as in (30), abstracting from a particular functional form,

$$dk = J(k) \tag{30}$$

Note that convergence will take place whenever $J'_K < 0$, what would be satisfied if investment depends negatively on the capital stock. Obviously, this is a relationship guarantied by the assumption of F''_{KK} less than zero.

Let us analyse next the dynamic evolution of income and savings, two fundamental variables in the characterisation of a country's external equilibrium.

3.5.2 Income Level and the Saving Rate Behaviour

From the standard production function specified in (18), it is determined an useful expression that, in terms of growth rates, is relating the volume of national production with its corresponding productive factors, the capital stock and labour force,

$$\hat{Y} = \frac{F'_K K}{Y}\hat{K} + \frac{F'_L L}{Y}n \tag{31}$$

Moreover, since in the case of constant returns to scale equation (26) is fulfilled, we can take advantage of this particular property to additionally simplify expression (31). In that case, finding $(F'_L L/Y)$ in (26) and substituting it in (31), this one can be finally rewritten as follows,

$$\hat{Y} = \hat{K} - \left(1 - \frac{F'_K K}{Y}\right)(\hat{K} - n) \tag{32}$$

This expression (32) is very useful indeed to conclude about the dynamic behaviour of the income growth rate over time. In particular, we observe that with $0 < (F'_K K/Y) < 1$ the relationship between $\hat{Y}$ and $\hat{K}$ will depend on the accumulation rate of the capital stock, that is, if it overpasses or not the level of *n*. Obviously, in equilibrium it is satisfied that $\hat{K} = \hat{Y} = n$.

Regarding the saving rate, we can assume a simple case where savings are proportional to the disposable income, as it appears in (33),

$$S = sY_d = s(Y - rD) \tag{33}$$

This way, the evolution of savings will depend on the evolution of income and on the level of

total foreign debt. Taking increments in (33) and dividing by S, we obtain expression (34) for the savings growth rate behaviour,

$$\hat{S} = \frac{\hat{Y} - (rD/Y)\hat{D}}{1-(rD/Y)} = \hat{Y} + \frac{(rD/Y)\hat{D}}{1-(rD/Y)}\left(\hat{Y} - \hat{D}\right) \tag{34}$$

In the first stages of growth, where D is sufficiently small, we find that $\hat{S} \cong \hat{Y} > n$. However, the empirical evidence shows that as foreign debt accumulates, the proportion of income that economies devote to the net payment of interests will be increasing over time. In terms of our model this would imply a per capita foreign debt growing faster than income, and a saving rate being lower that the income growth rate, i.e., $\hat{D} > \hat{Y} > \hat{S}$. In that case, only if the stock of foreign debt converges the economy will tend towards the steady state on its own. Most probably this steady state will be characterised by a situation where the national saving is insufficient to cover the investment needs of the economy.

However, we can consider instead another possibility; a situation where savings are large enough as for the stock of foreign debt to follow a decreasing process over time and the latter equilibrium conditions are eventually reversed. This dynamics opens the possibility for national savings to exceed investment necessities at a certain moment, so that the economy ends up accumulating foreign assets instead of debt. In that context, the steady state would have a considerable different characterisation.

Let us analyse next the specific conditions for the stock of foreign debt to converge.

3.5.3 Foreign Debt Convergence and the Possible Foreign Asset Accumulation

In order to analyse this process we start setting up in expression (35) the basic definition of the foreign debt growth rate,

$$\hat{D} = \frac{I - S}{D} \tag{35}$$

This one can be alternatively specified as in (36) in the case that the economy accumulates foreign debt, or as in (37) in the case that it accumulates foreign assets,

$$\hat{D} = rs + \frac{I - sY}{D} \tag{36}$$

$$\hat{F} = rs + \frac{sY - I}{F} \tag{37}$$

For the process to be stable, from (36) and (37) we need to check if $\Delta\hat{D}$ (or $\Delta\hat{F}$) tends to zero over time. An equation to characterise the $\Delta(\hat{D})$ dynamics is easily obtained from the original (35) formulation,

$$\ddot{A}(\hat{D}) = \left[\frac{\hat{I} - \hat{S}(S/I)}{1-(S/I)} - \hat{D}\right]\hat{D} \tag{38}$$

Moreover, substituting the $\hat{S}$ variable by its definition in (34), we easily get the following result,

$$\Delta(\hat{D}) = \frac{(\hat{I}-\hat{D})(1-rD/Y) - (S/I)(\hat{D}-\hat{Y})}{(1-S/I)(1-rD/Y)} \tag{39}$$

This equation (39) can be however additionally simplified by taking into account the savings definition given in (33). In particular, from this (33) expression, the following relationship will always occur,

$$\frac{rD}{Y} = 1 - \frac{S}{I}\frac{I}{sY} \tag{40}$$

Then, making use of equation (40), the previous (39) can be reformulated as follows,

$$\Delta(\hat{D}) = \frac{I(\hat{I}-\hat{D}) + sY(\hat{D}-\hat{Y})}{I-S} \tag{41}$$

As the aim is to analyse the convergence of the stock of foreign debt, it is determined next whether (41) would tend to zero over time. It is already known that as the diminishing returns of capital appear, the investment and income growth rates will progressively converge to *n*. In this respect, and in order to simplify, the conditions for convergence of the two dynamic variables of the problem, that is, the capital stock and the stock of foreign debt can be independently analysed. Provided then with the dynamic stability of investment, to examine (41) it is assumed that $\hat{I} = \hat{Y} = n$. In that case, (41) would reduce to the following expression,

$$\Delta(\hat{D}) = \frac{(\hat{D}-n)\ (sY-I)}{\Delta D} \tag{42}$$

which can be still simplified making use again of the savings definition. In particular, from (33) the next relationship can be also specified,

$$sY - I = (rs - \hat{D})D \tag{43}$$

So, taking into account the result in (43), equation (42) can be rewritten as follows,

$$\Delta(\hat{D}) = \frac{(\hat{D} - n)(rs - \hat{D})}{\hat{D}} \tag{44}$$

Or, alternatively, as in (45) in terms of foreign assets instead of foreign debt,

$$\Delta(\hat{F}) = \frac{(\hat{F} - n)(rs - \hat{F})}{\hat{F}} \tag{45}$$

From (44) and (45) it is clear that the convergence of the stock of foreign debt is not going to be so straightforward as in the case of investment but, on the contrary, it has to be carefully analysed. In particular, we will check that for the dynamics of the model, a key fact will be if the marginal savings variable *s* overpasses the level given by the ratio of *n* over *r*, i.e. n/r. Then, to conclude about this process we still need to look for some additional conditions that characterise the $\hat{D}$ dynamics[13].

Regarding the investment level we know that in equilibrium it will occur that $I = (n + \theta)K$. Hence, assuming the existence of constant returns to scale and that in the steady state $F'_K = r$, it would be true that[14] $I \leq (n/r)Y$. Moreover, taking into account the savings and foreign debt growth rate definitions given in expressions (33) and (35) respectively, we can finally get the following result,

$$\hat{D} - n \leq (n - rs)\left(\frac{Y - rD}{rD}\right) \tag{46}$$

Since $(Y - rD) \geq 0$, the sign of the last expression will exclusively depend on the relationship between *n* and *rs*. Therefore, given the result in (46), for the foreign debt to converge the only possibility is that $n > rs$. That is, the model will present dynamic stability only in the case that $s < n/r$.

On the other hand, from (36) and (37) it is easily obtained that the corresponding steady state values for the stocks of foreign debt and foreign assets are respectively determined by the following expressions,

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[13] Always given that investment and income have already reached their particular equilibrium levels.

[14] Note that with constant returns to scale $Y = F'_K K + F'_L L$. If we consider that $F'_K = r$, then $Y = rK + F'_L L$, so we have that $K \leq (Y/r)$.

$$D^* = \frac{I^* - sY^*}{n - rs} \tag{47}$$

$$F^* = \frac{sY^* - I^*}{n - rs} \tag{48}$$

where the star indicates the steady state position.

So, only if $s > (I/Y)^*$ the country will have the opportunity to end up as an asset holder. Let us simplify next the characteristics of this threshold condition.

In particular, we will assume a standard Cobb-Douglas production function that, in order to simplify, will not include technological progress. In this case, equation (18) can be specified in the following terms,

$$Y = K^{\alpha} L^{1-\alpha} \tag{49}$$

Or, alternatively as in (50), with variables expressed in per capita terms,

$$y = k^{\alpha} \tag{50}$$

Then, equation (47), again in per capita terms, can be reformulated as follows,

$$d^* = \frac{(n+\theta)(k^*)^{1-\alpha} - s}{n - rs} (k^*)^{\alpha} \tag{51}$$

The reader should note that (51) could be still simplified by determining the steady state level of the capital stock. In particular, we already know that in steady state the capital stock should fulfil that its marginal productivity equalises the real interest rate, that is, $F'_K(K,L) = r$. Therefore, taking into account (49), this would imply the following result for the equilibrium capital stock if expressed in per capita terms,

$$k^* = \left(\frac{\alpha}{r}\right)^{1/1-\alpha} \tag{52}$$

Substituting then (52) into (51) we finally get the next steady state foreign debt level in (53),

$$d^* = \frac{\alpha(n+\theta) - r\,s}{n - rs} \left(\frac{\alpha^{\alpha}}{r}\right)^{1/1-\alpha} \tag{53}$$

Alternatively, without loss of generality we can avoid the capital stock depreciation rate, so that (53) would simplify to the next (54) equation,

$$d^* = \frac{\alpha n - r s}{n - rs} \left(\frac{\alpha^{\alpha}}{r} \right)^{1/1-\alpha} \tag{54}$$

In that case the condition saying that $s > I/Y$ would simply converge to the one where $s > \alpha(n/r)$. We have found therefore the main conclusions of the model.

- **Corollary 1:** *In this model of small economy with the assumptions of no public sector, no technological progress and a non-depreciating capital stock, it will exist convergence to a long run stock of foreign debt only if* $s < n/r$.

- **Corollary 2:** *Given the stability condition, the economy would end up in a net debtor position in the case that* $s < a(n/r)$.

- **Corollary 3:** *Alternatively, if* $a(n/r) < s < n/r$ *the country will have the opportunity to end up as an asset holder.*

Taking into account these results we can use expressions (36), (37) and (44) to represent the different alternative dynamics of the system.

The high factor productivity of the first stages of growth will be undoubtedly characterised by a high investment rate. This level of investment together with the low accumulated debt would lead the country to a situation where the foreign debt grows at an important rate. This one should normally decrease as investment reaches its steady state position. In this respect, if the economy satisfies that $s < \alpha(n/r)$, investment will reach its equilibrium at a higher value than sY, i.e. $I^* > sY^*$. At that level, from (36) it is obtained that $\hat{D} > rs$, so in equation (44) it can be easily checked the dynamical stability of the process. This is the case when the economy gets to its steady state with an international net debtor position. Nevertheless, it is also true that the larger the value of the marginal savings, the smaller the final accumulated per capita foreign debt will be and, therefore, more likely that the national currency ends up at a more appreciated level.

Eventually, in the case that s overpasses the threshold of $\alpha(n/r)$, the decrease in the foreign debt growth rate will continue over time until a situation where the economy changes from having a net debtor position to a net creditor one. Naturally, this transition process will be characterised by a high rhythm of asset accumulation until it successively converges to its steady state position. Again, the dynamic stability of the process can be checked in equations (37) and (46) respectively. From (37) it is clear that when investment reaches its steady state position the stock of foreign assets will be growing over *rs*, so the system will require a local stability condition that is easily

derived from (46).

Thriftiness is therefore in this model a fundamental variable to characterise the growing process of the economy. Only if thriftiness is high enough, the economy will be able to move from a situation of net indebtedness, more likely in the first stages of growth, to a net creditor position. Of course, the real exchange rate will be directly influenced by this net foreign asset position.

This dynamic stability can be also proved through a rigorous analysis of the foreign debt dynamic equation. In particular expressions (55) and (56) give us the dynamic equilibrium of the stock of foreign debt in absolute levels and in per capita terms respectively,

$$d(D) = (I - S) \tag{55}$$

$$d(d) = i - (S/L) - nd \tag{56}$$

Substituting next the level of savings by its definition in (33), equation (56) can be also expressed as in (57),

$$d(d) = i - sy - (n - rs)d \tag{57}$$

Moreover, taking into account that both the investment and capital stock have reached their steady state levels, (57) will converge in equilibrium to equation (58),

$$d(d) = nk^* - s(k^*)^{\alpha} - (n - rs)d \tag{58}$$

And, if we substitute the steady state level of the per capita capital stock given in (52), the previous expression can be still reformulated as follows,

$$d(d) = \left(\frac{\alpha n}{r} - s\right)\left(\frac{\alpha}{r}\right)^{\alpha/1-\alpha} - \left(\frac{n}{r} - s\right)rd \tag{59}$$

From (59) it is clear that for the foreign debt -or net assets- stock to converge it is necessary that $s < (n/r)$, such as it was previously specified. Moreover, only in the case that $s > \alpha(n/r)$ the country will have the opportunity to end up as a foreign asset holder.

Next, charts from one to three show in graphical terms the alternative situations in which the economy would eventually converge to a net debtor position (Figure 1), to a net creditor position (Figure 2) or would follow a process of divergence (Figure 3).

Figure 1 Dynamics of Convergence in the Case of a Net Debtor Economy case $s < a\,(n/r)$

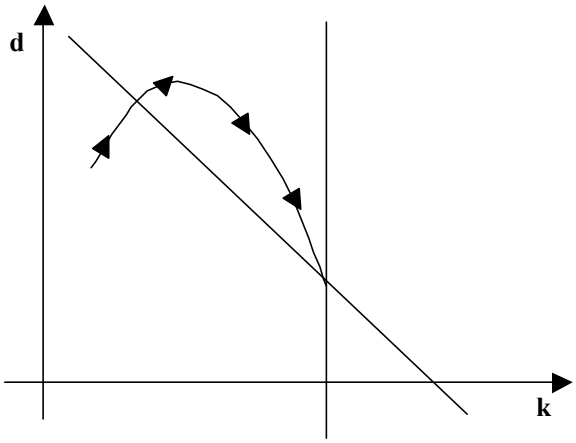


Figure 2 Dynamics of Convergence in the Case of a Net Creditor Economy case $a\,(n/r) < s < (n/r)$

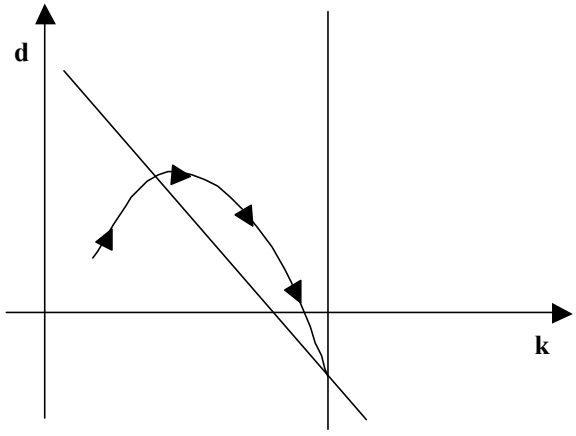
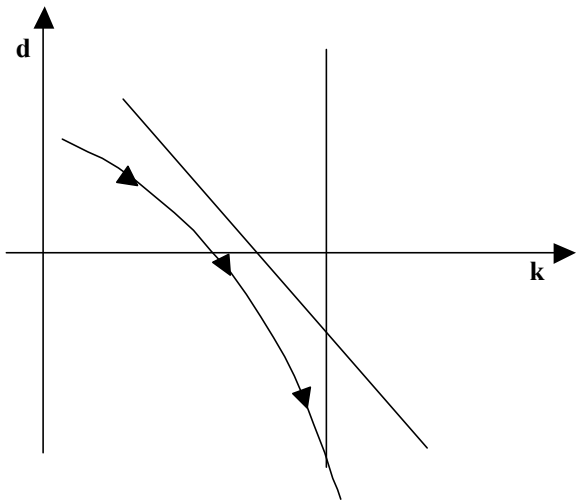


Figure 3 **Dynamics of Divergence**
s>(n/r)



3.5.4 The Real Exchange Rate and the Foreign Sector Path Towards Equilibrium

Finally, we will analyse in this section the implications of our previous results on the particular dynamics of the foreign sector.

The low level of income together with the high profitability of investment, commonly associated to the first stages of growth, is usually the cause by which the internal saving is not enough to cover the investment needs of an economy. In that case, it is usual that an international flow of resources from the surplus-producing countries to the loss-making ones appears, suffering these ones of increments in both the stock of foreign debt and the foreign interest payment. Notwithstanding, it is not habitual that the debt service of these first stages of growth overpasses the increment in foreign debt, so it should occur that the net transference of financial resources remains positive while the trade balance keeps on at negative values. To a great extent, the current account deficit is going to be due to the external capital inflow and not to the flow of interest payments.

From the current account and the trade balance definitions it is clear that their evolution will depend above all on the process of debt accumulation. Therefore, it is the steady state level of the foreign debt that will characterise the equilibrium of all, the current account, the trade balance and the real exchange rate variables. Let us analyse next this issue.

From (12) it is easy to obtain an expression for the trade balance growth rate,

$$\hat{TB} = \hat{D} - \frac{\Delta(\hat{D})}{r - \hat{D}} \tag{60}$$

Moreover, given the behaviour of debt provided in (42), the previous expressions can be also specified as follows,

$$\hat{TB} = \hat{D} - \frac{(\hat{D} - n)(rs - \hat{D})}{(r - \hat{D})\hat{D}} \tag{61}$$

Or, alternatively, as the symmetric equation (62),

$$\hat{TB} = \hat{F} - \frac{(\hat{F} - n)(rs - \hat{F})}{(r - \hat{F})\hat{F}} \tag{62}$$

From (61) and (62) it is clear that the trade balance would converge to a growth rate equal to *n* as the stock of foreign debt or foreign assets converges to its steady state level. The main differences will arise however in the absolute level of the trade balance position. In particular, the grater the accumulated stock of foreign debt, the greater the necessity of trade balance surplus for the economy to converge and, therefore, the more depreciated the real exchange rate will be in equilibrium. The idea underlying this result is that the trade balance should necessarily improve in order to finance the income outflow caused by the external indebtedness position.

Alternatively, in the case of asset accumulation, the process will lead the economy to a positive transference of resources that would eventually reverse the real exchange rate evolution.

In the first stages of repayment, the outflow of resources is going to be due not to the interest payment but to the external debt refund. In that case, and as foreign debt is repaid, the economy would accumulate assets until it ends up receiving positive financial flows. The process will necessarily result in a stronger national currency and a more deteriorated trade balance that, however, will keep on favouring the current account evolution.

3.5.5 The Real Exchange Rate, the Income Level and the Debt Cycle Theory

We will devote this section to summarise the main conclusions of our model on the theoretical relationship between income and real exchange rates over the different stages of growth. Curiously, our conclusions coincide broadly speaking with the idea of development proposed by the debt cycle theory. Specifically, we will highlight the three main characteristic periods in a common process of growth.

In a first stage, it will be usual that the volume of national savings cannot satisfy the high demand for investment. In that case, there will exist a large inflow of resources in the form of loans that will transitorily strengthen the national currency in real terms. Likewise, the economy would incur in important trade balance deficits accompanied in its turn by deficits on the current account.

In a second stage, the service of the net foreign debt will generate an international outflow of resources that at a certain moment would start to overpass the volume of resources coming from abroad. Sooner or later, this change provoked by the financing of the growth process will end up generating a weaker national currency. Naturally, the eventual result is a trade balance surplus, given the necessity of serving the debt, and a current account deficit.

Potentially, the economy will be able to enter in a new stage in which the total amount of debt is repaid and there is even an asset accumulation. In that case, there will be a net inflow of resources that, in the long run, would guarantee a more appreciated national currency and, indeed, a current account surplus. The result is therefore that, as long as the three stages of growth are covered, high income levels will be observed along with stronger national currencies. Only in some particular circumstances the economy is not going to acquire a creditor position, so it would be characterised as in the second stage of growth. Unfortunately, the economy will not be able to overcome in this case the initial weakness of its national currency.

Nevertheless, it is reasonable to think that although with low income levels consumption has a strong weight in the disposable income composition, as economies access to more resources a sensible evolution is that agents increase their thriftiness parameter[15]. Therefore, it will not be strange to observe stronger national currencies in economies with higher income levels. Exceptions to this rule will be those countries that have got into an excessive indebtedness position in their growing process.

4. Summary and Concluding Remarks

The empirical evidence shows that associated with the growing process of an economy, there is normally a process of loss of competitiveness against less developed countries. This evidence is what has contributed in fact to the development of the branch of literature that rejects PPP accomplishment and look for alternative explanations that highlight the role of real exchange rate fundamentals.

Specifically, some different theories have been proposed in order to explain the positive correlation between the real exchange rate and the per capita gross domestic product. These theories concentrate on the case of an economy with two sectors, tradables and nontradables, and

[15] Note that this analysis does not consider agents thriftiness evolution over time.

emphasise the role of GDP in the determination of the relative price level. The traditional theories under this approach are mainly supply-oriented models like the Balassa-Samuelson hypothesis or the Heckscher-Ohlin model. As opposed to these theories, demand-oriented approaches have been incorporated afterwards.

However, the main problem of these theoretical attempts is that they have not considered the financial restrictions imposed by the course of the economic growth. Thereby, for a comprehensive analysis of this economic problem under debate, it is also important to conclude about the intertemporal aspects that the financing of the growth process necessarily imposes. That is, in this context the equilibrium condition of the balance of payments should be also taken into account.

We highlight next the major conclusions of the paper.

We can distinguish three differentiated stages in the growth process of a standard economy. In a first stage, it will occur that large inflows of external resources would transitorily appreciate the national currency. However, in a second stage, the appearance of the diminishing returns of capital will halt investment growth and, therefore, the need for external financing. In this case, a steady state can be reached where the economy held a net indebtedness position. Occasionally, in a third stage, if thriftiness is high enough, a new equilibrium can occur where the country held a net creditor position, and where there would be a net inflow of resources strengthening the national currency.

Nevertheless, if we take into account that as economies grow agents thriftiness can increase, it can be usual to observe, independently of the net indebtedness or creditor position, that the higher the income level of an economy, the more appreciated the real exchange rate will be. Exceptions to this rule will be those countries that have got into an excessive debt accumulation.

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